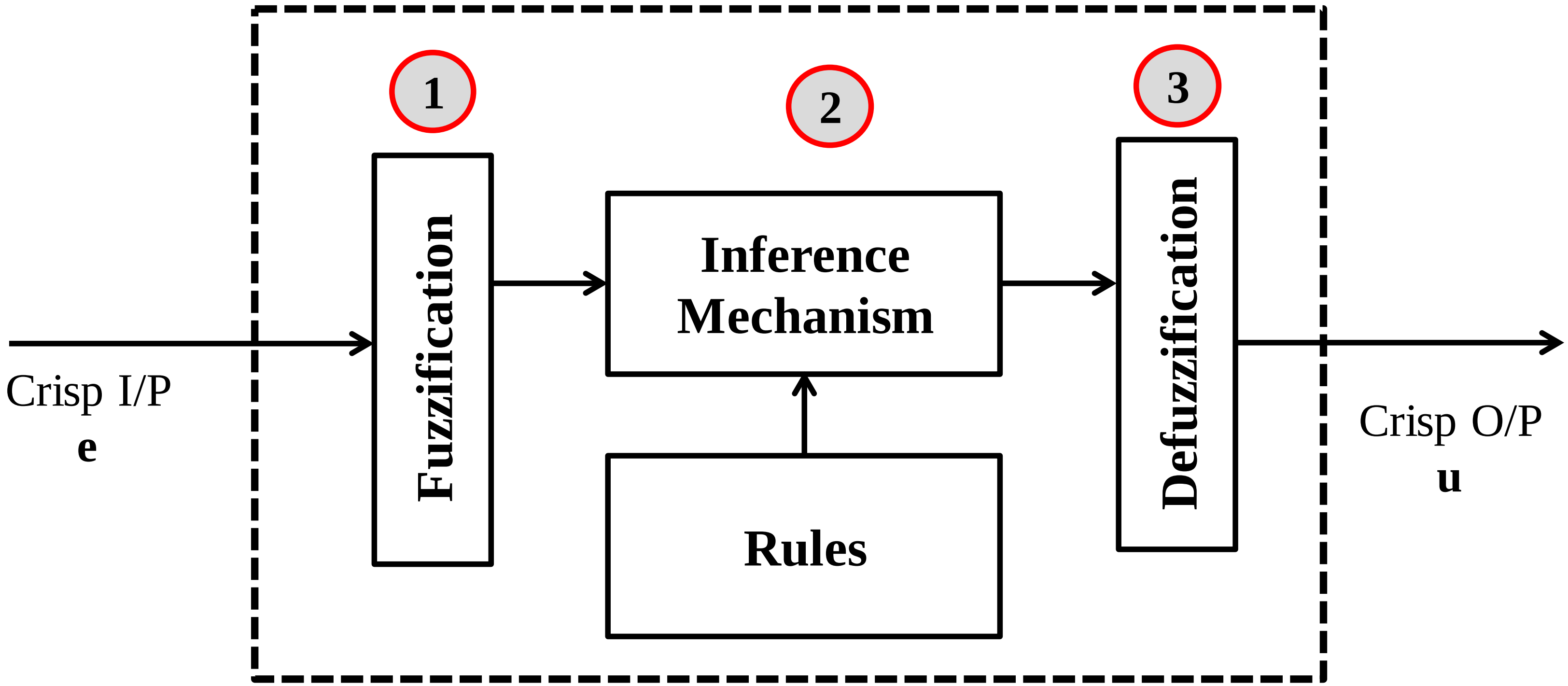
بسم الله الرحمن الرحيم

Fuzzy Control Course

Lec 6

Fuzzy Controller Structure



Fuzzy Controller

- Crisp means numeric (or real) value

Fuzzy Inference System (Controller)

- Inference is the process of formulating a nonlinear mapping from a given crisp input to a crisp output. The mapping then provides a basis from which decisions can be made. The process of fuzzy inference involves all the membership functions (fuzzy sets), operators and IF–THEN rules.
- **There are three types of fuzzy inference** which have been widely employed in various fuzzy systems and applications. These fuzzy inferences (models/controllers) are: **(1) Mamdani fuzzy inference**

  (2) Sugeno fuzzy inference

  (3) Tsukamoto fuzzy inference

  The most commonly-used are Mamdani-type and Sugeno-type.

(1) Mamdani Fuzzy Inference

- The Mamdani-type fuzzy inference was first proposed as an attempt to control a steam engine and boiler using a set of linguistic control rules obtained from an experienced human operator.
- **EX:** To illustrate the Mamdani-type fuzzy mechanism, consider a two input single-output Mamdani fuzzy model, each input x_1 , x_2 and output y has two MFs:{A1, A2}, {B1, B2}and{C1, C2}, respectively. If we consider two rules R_1 and R_2, these rules are:

$$R_1: \text{IF } x_1 \text{ is } A1 \text{ AND } x_2 \text{ is } B1 \text{ THEN } y \text{ is } C_1$$

$$R_2: \text{IF } x_1 \text{ is } A2 \text{ AND } x_2 \text{ is } B2 \text{ THEN } y \text{ is } C_2$$

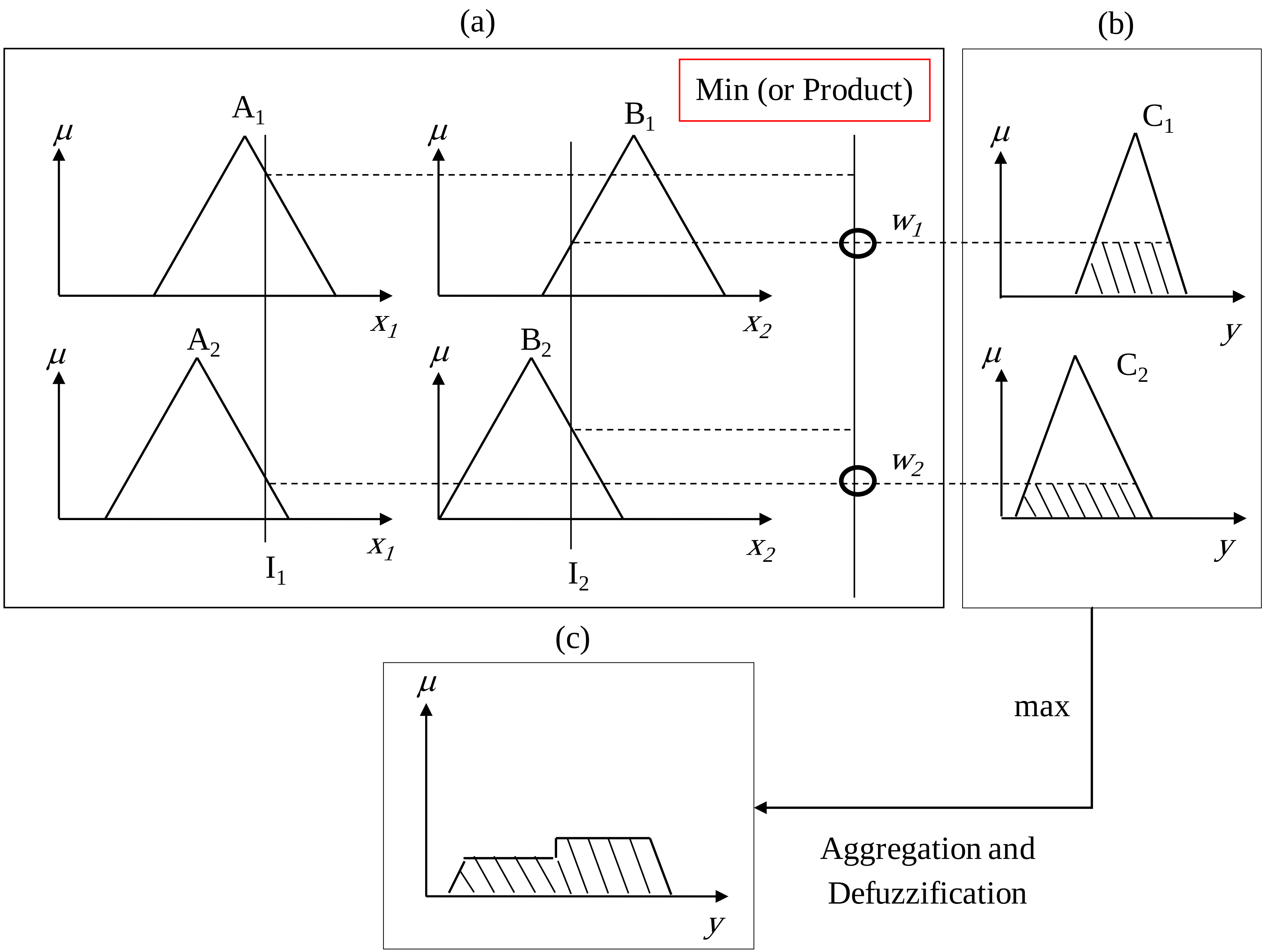


Fig. 1 Two-input single-output Mamdani fuzzy model

(1) Mamdani Fuzzy Inference

- In the Mamdani -type fuzzy model shown in Fig.1, two measured crisp values I_1 and I_2 are used for the inputs x_1 and x_2, respectively.

- Fig.1-a shows the fuzzification and inferencing using the minimum rule (AND operator in the IF-Part can be represented using Min. or product rule) for computing the firing strengths w_1 and w_2 for the premise terms of the rules.

$$w_1 = \min(\mu_{A1}, \mu_{B1}) \quad , \quad w_2 = \min(\mu_{A2}, \mu_{B2})$$

or

$$w_1 = \mu_{A1} \cdot \mu_{B1} \quad , \quad w_2 = \mu_{A2} \cdot \mu_{B2}$$

- w_1 and w_2 are stand for $\mu_{Premise1}$ and $\mu_{Premise2}$ as we used before, these weights represent the strengths for the firing rules.

(1) Mamdani Fuzzy Inference

- The inferred output of each rule is the truncated membership functions chosen from the minimum firing strength as shown in Fig.1-b. The truncated membership functions for each rule are aggregated as shown in Fig.1-c and any of the following defuzzification methods (like Center of gravity (COG)) is carried out to convert a fuzzy set to a crisp value, these methods are:
- Center of Gravity (COG) method
- Weighted average method
- Mean-max membership method

(2) Sugeno Fuzzy Inference / TSK Fuzzy Inference

- The Sugeno fuzzy inference, also known as the TSK fuzzy model, was proposed by Takagi, Sugeno and Kang in 1985.
- The output of each rule of the fuzzy IF-THEN rules (consequent or then part) is a linear function which is a combination of input variables plus a constant term.
- **EX:** To illustrate the Sugeno-type inferencing mechanism, consider a two- input single-output TSK fuzzy model, each input x_1 and x_2 has two membership functions (MFs) $\{A1\ ,\ A2\}$ and $\{B1\ ,\ B2\}$, if we consider two rules R_1 and R_2 with consequent functions $\{y_1\ ,\ y_2\}$.These rules are:

R_1 : IF x_1 is $A1$ AND x_2 is $B1$ THEN $y_1 = p_1 x_1 + q_1 x_2 + r_1$

R_2 : IF x_1 is $A2$ AND x_2 is $B2$ THEN $y_2 = p_2 x_1 + q_2 x_2 + r_2$

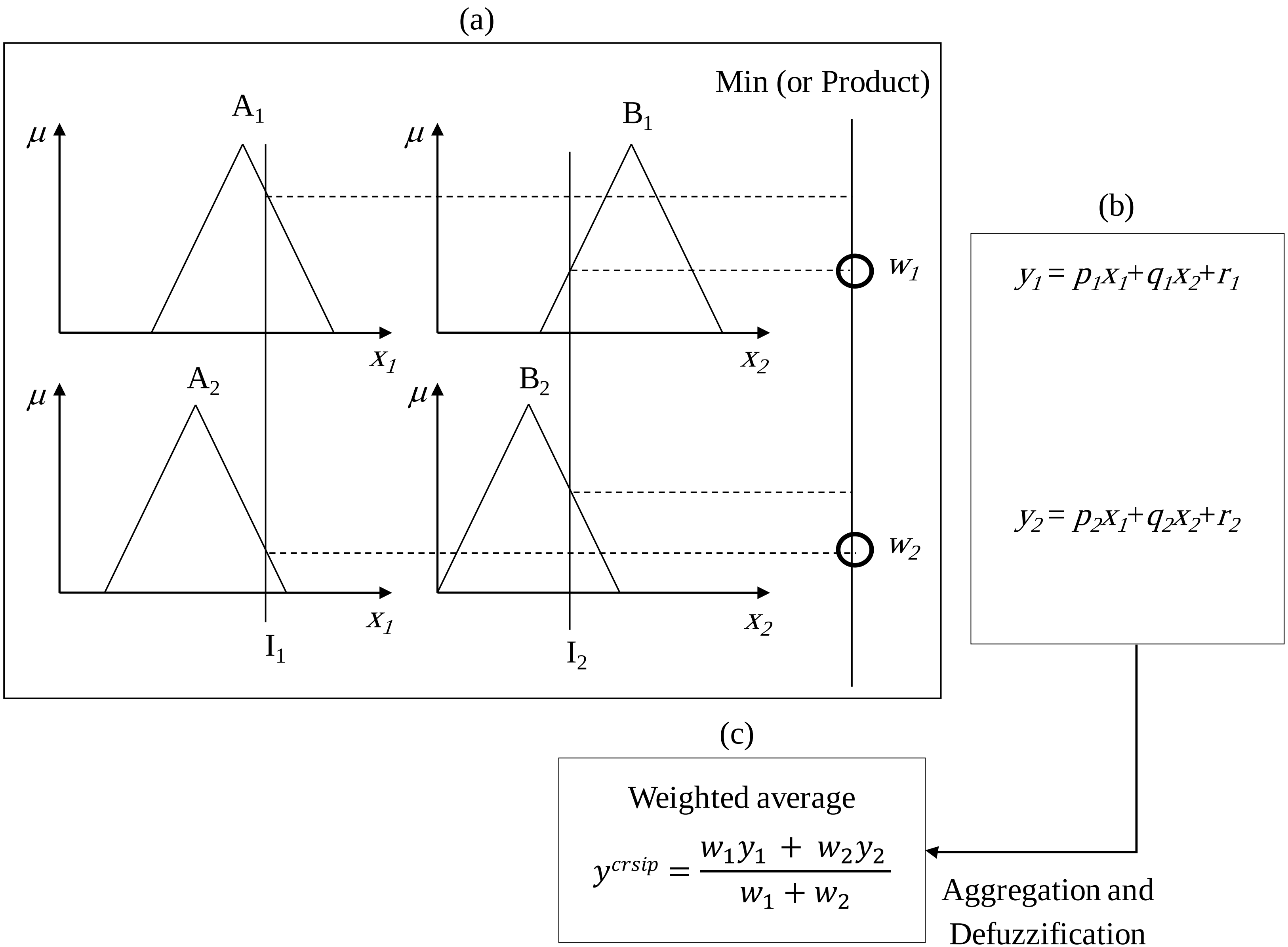


Fig. 2 Two-input single-output TSK fuzzy model

(2) Sugeno Fuzzy Inference / TSK Fuzzy Inference

- In the Sugeno-type fuzzy model shown in Fig.2, two measured crisp values I_1 and I_2 are used for the inputs x_1 and x_2, respectively.
- Fig. 2-a shows the fuzzification and inferencing using the minimum or product rule for computing the firing strengths w_1 and w_2 for the premise term of the rules. The firing strength is calculated using the minimum or product rule as:

$$w_1 = \min(\mu_{A1}, \mu_{B1}) \quad , \quad w_2 = \min(\mu_{A2}, \mu_{B2})$$

or

$$w_1 = \mu_{A1} \cdot \mu_{B1} \quad , \quad w_2 = \mu_{A2} \cdot \mu_{B2}$$

- w_1 and w_2 are stand for $\mu_{Premise1}$ and $\mu_{Premise2}$ as we used before, these weights represent the strengths for the firing rules.

(2) Sugeno Fuzzy Inference / TSK Fuzzy Inference

- Once the parameters $\{p_1, q_1, r_1, p_2, q_2, r_2\}$ are known, the consequent y_1 and y_2 are calculated for each rule using a first-order polynomial as shown in fig.2-b. The overall output y is obtained via the weighted average of the crisp outputs y_1 and y_2. The weighted average defuzzification method is computed by:

$$y^{\text{crisp}} = \frac{w_1 y_1 + w_2 y_2}{w_1 + w_2}$$

- Fig.2-c illustrates the aggregation and final defuzzified value for the TSK model.

(3) Tsukamoto Fuzzy Inference

- In the Tsukamoto fuzzy inference, the consequent of each fuzzy if–then rule is represented by a monotonic MF (function between ordered sets that preserves the given order).
- **EX:** To illustrate the Tsukamoto-type mechanism, consider a two input single-output Tsukamoto fuzzy model, each input x_1 and x_2 has two membership functions (MFs) $\{A1, A2\}$ and $\{B1, B2\}$, if we consider two rules R_1 and R_2 with consequent monotonic functions $\{C_1, C_2\}$. These rules are:

R_1: IF x_1 is $A1$ AND x_2 is $B1$ THEN y is C_1

R_2: IF x_1 is $A2$ AND x_2 is $B2$ THEN y is C_2

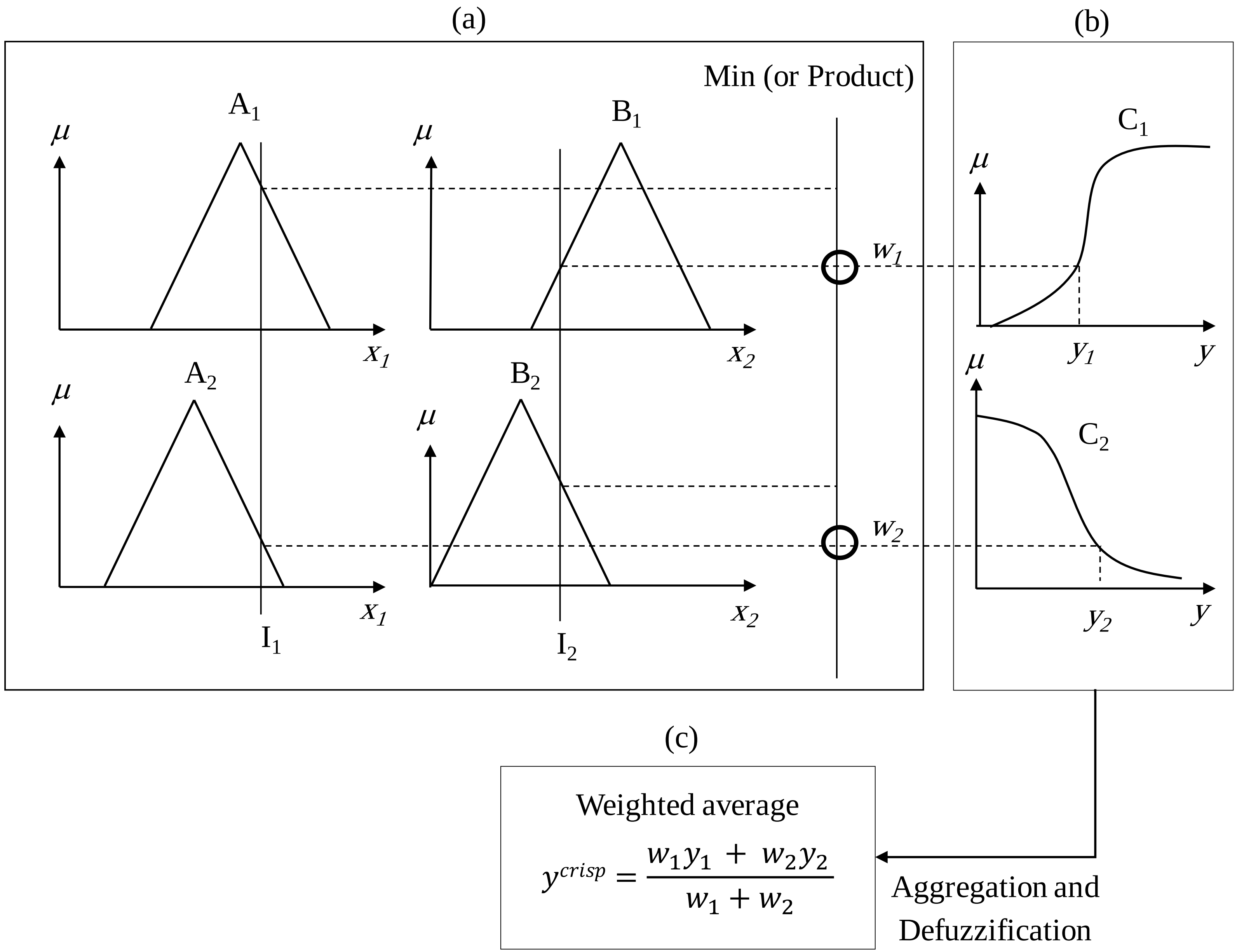


Fig. 3 Two-input single-output Tsukamoto fuzzy model

(3) Tsukamoto Fuzzy Inference

- In the Tsukamoto-type fuzzy model shown in Fig.3, two measured crisp values I_1 and I_2 are used for the inputs x_1 and x_2, respectively.
- Fig.3-a shows the fuzzification and inferencing using the minimum or product rule for computing the firing strengths w_1 and w_2 for the premise term of the rules. The firing strength is calculated using the minimum or product rule.
- The consequent y_1 and y_2 represent the defuzzified outputs (one value for each rule) and are determined as shown in fig.3-b.
- The overall output y is obtained via the weighted average of the crisp outputs y_1 and y_2.

(3) Tsukamoto Fuzzy Inference

- The overall output y is obtained via the weighted average of the crisp outputs y_1 and y_2. The weighted average defuzzification method is computed by:

$$y^{\text{crisp}} = \frac{w_1 y_1 + w_2 y_2}{w_1 + w_2}$$

- Fig.3-c illustrates the defuzzified value for the Tsukamoto model.

- **Despite the simplification of the defuzzification procedure, the Tsukamoto fuzzy model is not used very often.**

Example: Steam Engine System Using TSK-fuzzy Model

- A steam engine system has been developed using a Takagi–Sugeno fuzzy model, where x_1 represents speed, x_2 represents pressure and y represents throttle position.
- Two membership functions for speed x_1 and pressure x_2, defined within the same universe of discourse [0, 10], are shown in Fig. 4. For the linguistic variable x_1, the MFs are taken to be $\{A1, A2\}$ and for x_2, the MFs are taken to be $\{B1, B2\}$. The throttle position y is defined by the four first-order polynomial functions below:

$$
\begin{aligned}
y_1 &= 3x_1 + 2x_2 + 1 \\
y_2 &= x_1 + 3x_2 + 1 \\
y_3 &= x_1 + 2x_2 \\
y_4 &= 2x_1 + 5
\end{aligned}
$$

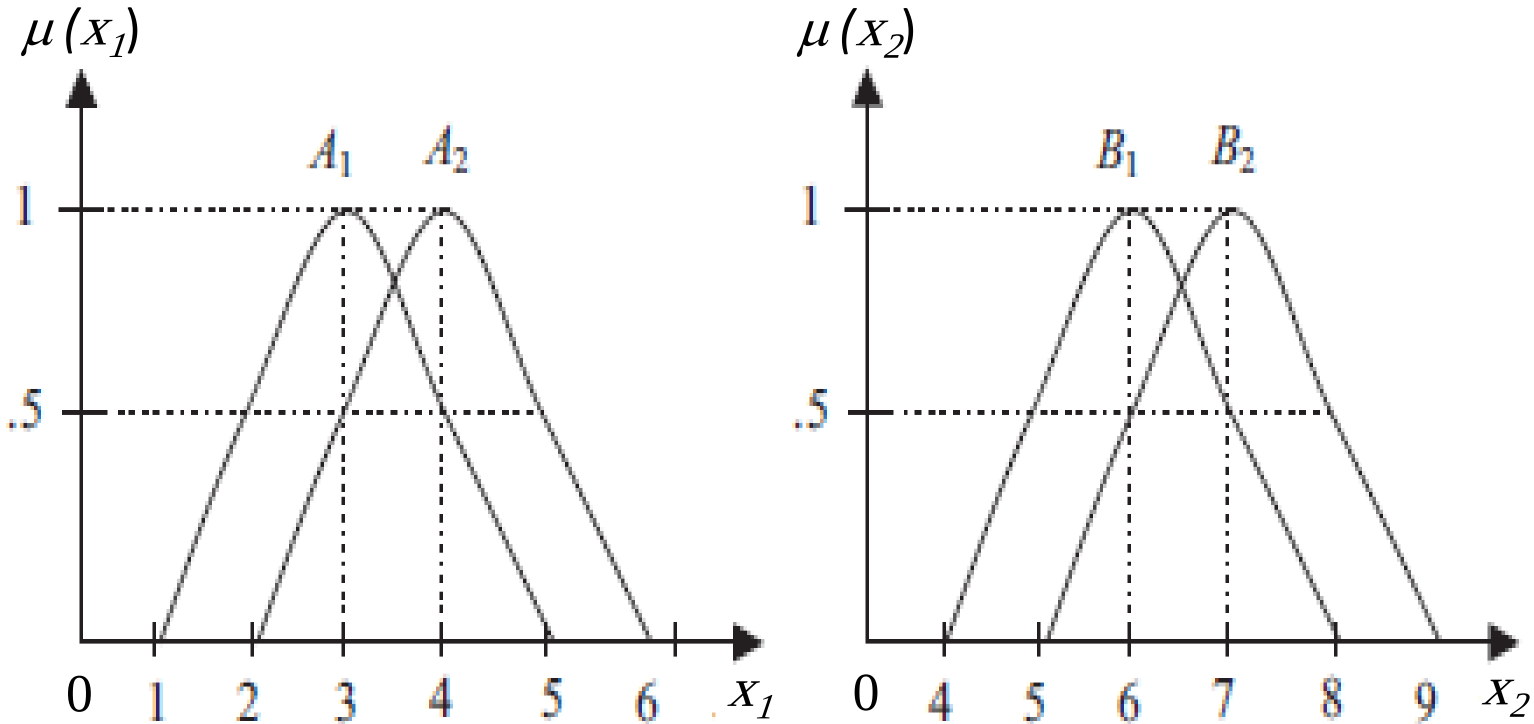


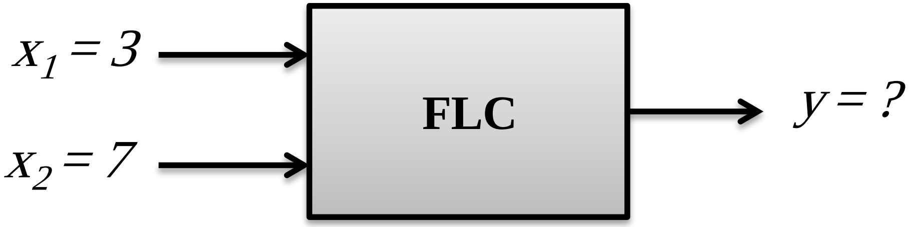
Fig. 4: MFs for inputs x_1 and x_2

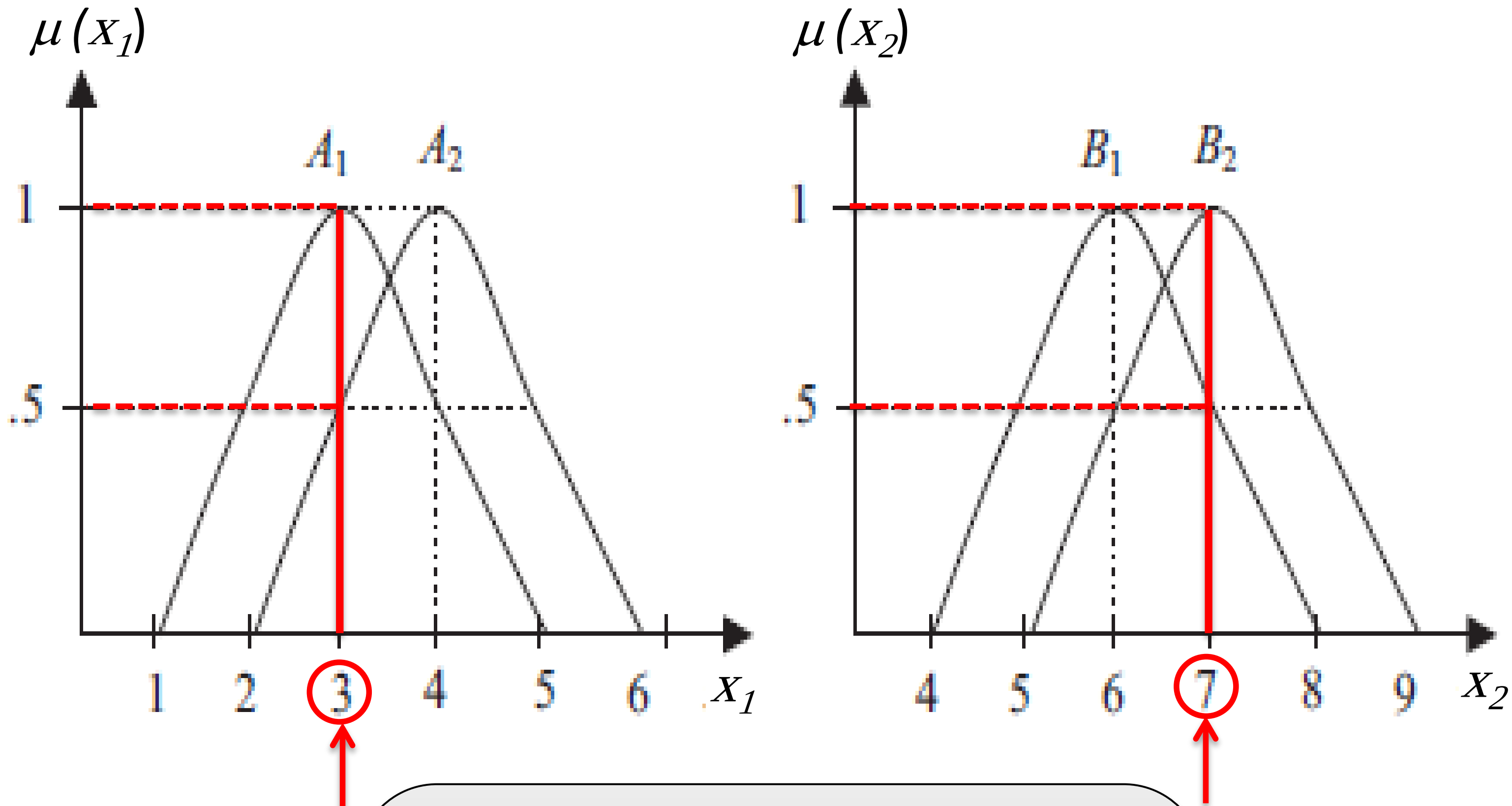
Example: Steam Engine System Using TSK-fuzzy Model

- The rule base of the TSK-fuzzy model consists of four rules, as shown in the following table:

x_1 \ x_2	$B1$	$B2$
$A1$	y_1	y_2
$A2$	y_3	y_4

- **Find the throttle position y, if the inputs take values $x_1 = 3$ and $x_2 = 7$.**





1- Fuzzification

x_1 is $A1$ with $\mu_{A1}(x_1 = 3) = 1$

x_1 is $A2$ with $\mu_{A2}(x_1 = 3) = 0.5$

x_2 is $B1$ with $\mu_{B1}(x_2 = 7) = 0.5$

x_2 is $B2$ with $\mu_{B2}(x_2 = 7) = 1$

2- The Fired Rules:

R1: IF x_1 is $A1$ AND x_2 is $B1$ THEN $y_1 = 3x_1 + 2x_2 + 1$

R2: IF x_1 is $A1$ AND x_2 is $B2$ THEN $y_2 = x_1 + 3x_2 + 1$

R3: IF x_1 is $A2$ AND x_2 is $B1$ THEN $y_3 = x_1 + 2x_2$

R4: IF x_1 is $A2$ AND x_2 is $B2$ THEN $y_4 = 2x_1 + 5$

Where at $x_1 = 3$, $x_2 = 7$:

$y_1 = 3*3 + 2*7 + 1 = 24$

$y_2 = 3 + 3*7 + 1 = 25$

$y_3 = 3 + 2*7 = 17$

$y_4 = 2*3 + 5 = 11$

2- The Fired Rules:

R1: IF x_1 is $A1$ AND x_2 is $B1$ THEN $y_1 = 24$

R2: IF x_1 is $A1$ AND x_2 is $B2$ THEN $y_2 = 25$

R3: IF x_1 is $A2$ AND x_2 is $B1$ THEN $y_3 = 17$

R4: IF x_1 is $A2$ AND x_2 is $B2$ THEN $y_4 = 11$

3- The strength of the fired rules:

R1: $w_1 = \mu_{premise1} = \min \{ \mu_{A1}(x_1) , \mu_{B1}(x_2) \} = \min \{1 , 0.5\} = 0.5$

R2: $w_2 = \mu_{premise2} = \min \{ \mu_{A1}(x_1) , \mu_{B2}(x_2) \} = \min \{1 , 1\} = 1$

R3: $w_3 = \mu_{premise3} = \min \{ \mu_{A2}(x_1) , \mu_{B1}(x_2) \} = \min \{0.5 , 0.5\} = 0.5$

R4: $w_4 = \mu_{premise4} = \min \{ \mu_{A2}(x_1) , \mu_{B2}(x_2) \} = \min \{0.5 , 1\} = 0.5$

4- Aggregation and Defuzzification:

R1: $y_1 = 24$ with $w_1 = 0.5$

R2: $y_2 = 25$ with $w_2 = 1$

R3: $y_3 = 17$ with $w_3 = 0.5$

R4: $y_4 = 11$ with $w_4 = 0.5$

Using weighted average method

$$y^{crisp} = \frac{w_1 y_1 + w_2 y_2 + w_3 y_3 + w_4 y_4}{w_1 + w_2 + w_3 + w_4}$$

$$y^{crisp} = \frac{0.5 * 24 + 1 * 25 + 0.5 * 17 + 0.5 * 11}{0.5 + 1 + 0.5 + 0.5}$$

$$y^{crisp} = 20.4$$